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VULNERABILITY PARAMETERIZATION FOR MASONRY CHIMNEYS BY MEANS OF A LIMIT ANALYSIS APPROACH

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Abstract. *The paper gives a quantitative insight into the vulnerability of existing masonry chimneys located in Spain, accounting for horizontal loads, geometry and material deterioration. During the past two centuries, the industrialization process gave rise to a quick increase in the construction of chimneys, which were made almost exclusively by brickwork but present an almost infinite variety of geometries. Nowadays such structures are not used anymore as industrial artifacts, but they belong to the historical heritage and are constantly subjected to natural hazards. To their conservation, a high priority exists, because their specificities and their complex structural behavior are unique. A large survey of chimneys in terms of geometry, possible pre-existing damages, and their location has been carried out in the past. The parameterization of their vulnerability is here carried using a kinematic limit analysis approach based on different pre-assigned failure mechanisms. It is observed an inverse proportional relationship among slenderness, present damage parameters, and the expected lateral load-bearing capacity. The parameterization here proposed permits to characterize in a fast and reliable way the vulnerability of such structures and to propose further actions aimed at an effective strengthening.*

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1 INTRODUCTION

The effects on the built stock of an earthquake change from one structure to another and they are often very difficult to be predicted. Therefore, it is imperative that seismic vulnerability investigations are carried out case by case. As for all existing masonry structures, it is proved that historical ones are particularly vulnerable to seismic actions [1–3]. Among the different typologies of existing historical masonry structures, slender ones are very common, as for instance chimneys and some towers. In this study we will focus on masonry chimneys, particularly those located in Spain, for which a comprehensive geometric survey is already at disposal [4].

The diffusion of masonry chimneys was strongly associated to industrial processes spreading and their architecture was obviously influenced by the British Victorian Architecture [4]. At the very beginning, chimneys were constructed with masonry, in stones or bricks, i.e. using the natural materials available in the neighborhood. Nowadays, commonly used materials are reinforced concrete and steel, allowing to achieve considerable heights that would be impossible with masonry. In the literature, it is possible to find different researches addressing issues like structural damages of chimneys [5–7], structural response to natural hazards [7, 8], and retrofitting possibilities [9, 10].

The aim of this study is to propose a simple numerical tool for the seismic vulnerability evaluation of masonry chimneys. To maintain the approach sufficiently simple to be used by common engineers, a limit analysis approach is adopted. A similar approach has been recently applied also on a masonry chimney for a fast investigation of its load carrying capacity against horizontal loads [12], adopting the criteria recommended by the Italian Code [13]. In addition, this study is intended as preliminary reference for an extension of the problem to a variety of geometries and possible failures under horizontal loads. Throughout a deep investigation carried out by one of the authors on the architectural features of several masonry chimneys in Spain [14], a representative geometry variable in dependence of some few parameters is identified.

From the study, it is concluded that a straightforward relationship between the chimney structure and its expected failure is difficult to obtain, because of the almost infinite variability of the geometry and the mechanical properties of the masonry material. As a consequence Monte Carlo MC simulations are carried out to obtain different scenarios in terms of active failure mechanisms obtained and corresponding accelerations at collapse. Large-scale MC simulations are performed using a manual limit analysis approach, i.e. assuming the collapse of a chimney possible only for the formation of a limited set of pre-assigned mechanisms. The results obtained draw important conclusions on the expected vulnerability of a real masonry chimney, and the corresponding collapse multiplier. The obtained results have a practical value, because a fast and reliable assessment of the vulnerability is possible without the need to perform non-linear FE analyses.

2 GENERAL DESCRIPTION OF MASONRY CHIMNEYS

A chimney is constituted generally by three distinct elements: the base, the shaft, and the crown. The foundations are deliberately not considered, because they are generally stiffer than the other parts. Therefore, in the present study, the base is assumed fixed. Another assumption made to simplify the problem is related to the flue pipe opening. Normally its dimensions are very small [15], and it is not expected to affect the response of the chimney subjected to lateral loads significantly.

The Base: - The cross-section of the base is typically square, despite the cross-section of the shaft may be generally circular or octagonal. Commonly, it is built with bricks, but it can also be found completely built in ashlar. Sometimes the base was altered between bricks and ashlar at the corners. These considerations could make us believe that a better material is expected in this zone. The typical dimensions of the base height vary from 3 to 5 m. Some illustrative examples of bases from Spanish masonry chimneys are depicted in Figure 1.

Figure 1: Different configurations of chimney's base taken from masonry chimneys in Spain, [14].

The Shaft: - This is the most important and functional part of a chimney, making possible the expelling process of the produced industrial fumes. The cross-section of the shaft, which may differ from the cross-section of the base, characterizes the type of the chimney. Four typical shapes are used to build the cross-section of the shaft: square, circle, hexagon or octagon, see Figure 2. The shape of the shaft is related to aerodynamical effects, but mostly it demonstrates the construction tradition of a specific zone. The outer diameter of the shaft usually decreases, following the slope of the shaft that takes a value between 1% and 3%. The thickness of the shaft wall does not remain constant through height, but typically it reduces by one brick width (12cm) every 4÷6m height [14].

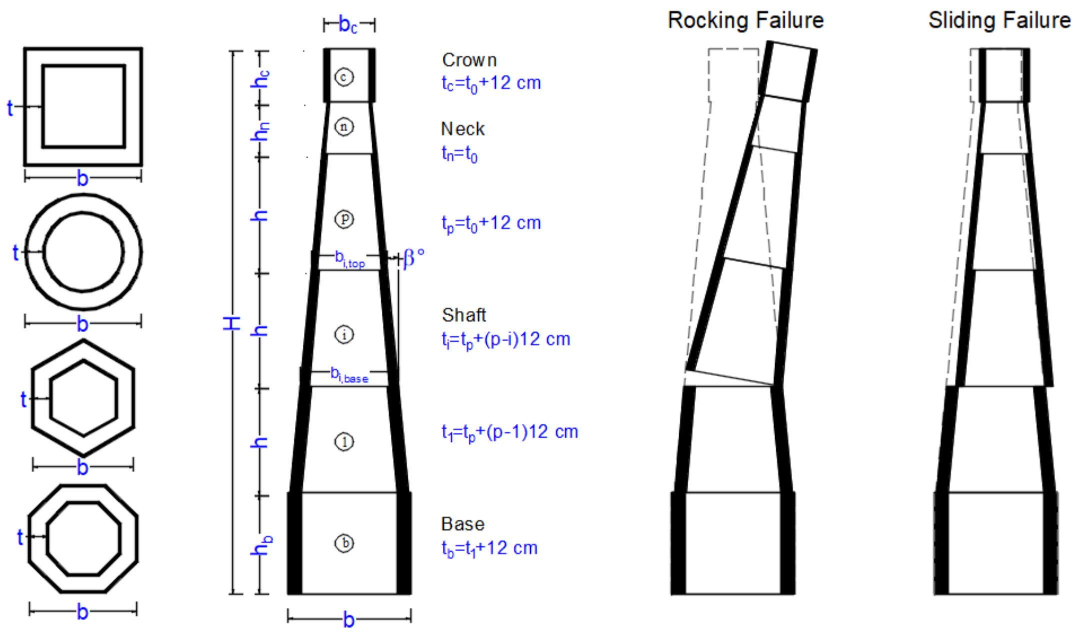
Figure 2: Different examples of chimney's shaft taken from masonry chimneys in Spain, [14].

The Crown: - It is an extension of the stack with a main ornamental function aimed to attract the attention. These decorations, mostly in combination with the aerodynamical aspects lead to a slight enlargement of the section and weight of the crown. Some illustrative examples of crowns are depicted in Figure 3.

Figure 3: Different types of crowns taken from masonry chimneys in Spain, [14].

3 PARAMETRIZATION OF THE CHIMNEY'S GEOMETRY

Basing on hundreds of observations on real chimneys in Spain, a simple parametrization of their geometry that can be used for further investigations is carried out. The regularity of the structural typology provides a tool to reduce the parameters required to describe a generic chimney.



a) b)

Figure 4: a) The simplified geometry of a masonry chimney, b) Two typical failure mechanisms.

Many chimney bases are narrowed with a slope higher than the slope of the shaft, but in this study, it is considered with a prismatic shape. This approach, as it will be demonstrated later does not affect the vulnerability assessment of chimneys. The shaft is discretized into p blocks with the same height (4÷6m) but with a difference in the base and the top area of each block, attributed to the conical/pyramidal shape of a typical chimney. The upper block of the shaft, herein called the neck, is the thinnest part of the shaft, which normally has a smaller height than the other blocks below. Normally the wall thickness of the neck is 12 cm (one longitudinal row of bricks), but in some cases this can exceed 24 cm. The macro-blocks below the neck increase typically their thickness as depicted in Figure 4. The reduction of the cross-section wall thickness, moving from the base to the crown, decreases the weight of the chimney, consequently having a positive contribution against seismic actions, but on the other hand, the cross-section and hence the resisting bending moment is decreased (also because the weight decreases). It is supposed that such sections, subjected to stress concentration, are probably subjected to crack initiation and further propagation induced by lateral acting forces, see Figure 4-b). A special comment could be drawn regarding the possible sliding failure at a certain level of the masonry chimney. Some recent researches related to the failure investigation of masonry towers by means of simplified approaches [15, 16], have shown that this failure is not a typical failure that could often occur in slender structures, unless special conditions are met. These conditions are related to the failure criteria, where a typical failure criterion adopted for masonry is the Mohr-Coulomb one, i.e. where the shear strength is mostly to the level of vertical compressive and the friction angle [17, 18]. The influence of the material quality is investigated for what regards the possibility of preventing or activating a certain failure mechanism and the corresponding load-bearing capacity of the structure.

This study encompasses a wide range of geometrical parameters (represented in Figure 2) basing on comprehensive surveys carried out on Spanish masonry chimneys. In the following table, it is summarized the range of these parameters, which will be uniformly generated at randon, in the absence of a full statistical database of their actual statistical distribution.

Parameter	Parameter bandwidth	Parameter	Parameter bandwidth
H	$10 \div 50\,m$	h_c	$1 \div 3\,m$
λ	$\lambda_o - 3 \div \lambda_o + 3$	h	$5, 6\ or\ 7\,m$
β	$1.5° \div 3°$	t_0	$12\ or\ 24\,cm$
h_b	$3 \div 6\,m$		

Parameter	Value	Parameter	Value
b	H/λ	b_i	$b_c - 0.24 + 2(h_n + i\,h)\tan\beta$
h_n	$H - h_b - h_c - ph$	b_c	$b - 2(H - h_b - h_c)\tan\beta + 0.24$
p	$int(\,(H - h_b - h_c)/h)$		

Table 1: Characteristic values of the chimneys parameters.

In the table, $\lambda_o$is the mean value of the slenderness, given with this equation: $\lambda_o = 0.0897\,H + 8$ and derived by a linear regression analysis based on the data provided in [14].

4 LIMIT ANALYSIS APPROACH

Limit analysis of masonry structures is becoming a very powerful tool for fast and reliable analyses even in presence of seismic loads. Since the pioneering work by Heyman [20], further studies have successfully adopted and developed this technique. This approach has been employed also in combination with FEM by one of the authors, who studied also complex existing masonry structures like palaces, churches or towers [20–22]. Recently, the approach of NTM has been modified adding a limited tensile strength, which tends to alleviate the typical underestimation of the overall structural capacity to resist horizontal loads exhibited by NTM. A limited tensile strength under rocking failure is here accounted for, as can be seen in Figure 5.

Similar to chimneys, masonry towers have been recently investigated by means of a simple upper bound limit analysis approach, assuming possible five different failure mechanisms observed during the past earthquakes [16]. In the absence of an equivalent data set of observed damages during past earthquakes in masonry chimneys, a simplified approach is developed here basing on the failure mechanisms described in Figure 4-b). A more detailed scheme is depicted in Figure 5. In such a figure, failure corresponding to pure shear (as depicted in Figure 4-b) is not shown for the sake of conciseness. However, such a failure is taken into consideration in the computations, observing that its occurrence is mostly related to material properties degradation. The implementation done is in agreement with recommendations of the Italian Code for towers [13].

Due to the parametrization of the structure geometry, we can distinguish four main possible failure mechanisms, respectively with hinged located at the crown, the neck, the base and in one of the sections of the shaft, for rocking or shear failures respectively. We classically apply the principle of virtual powers to each failure mechanism. Assuming as P_k the weight of the block, $\delta_{x,k}$, $\delta_{y,k}$ virtual displacements, L_{fi} the virtual work done by internal actions, the collapse load multiplier α_0 can be therefore estimated as follows:

$$\alpha_0 \sum_{k=1}^{n} P_k \delta_{x,k} - \sum_{k=1}^{n} P_k \delta_{y,k} = L_{fi} \quad (1)$$

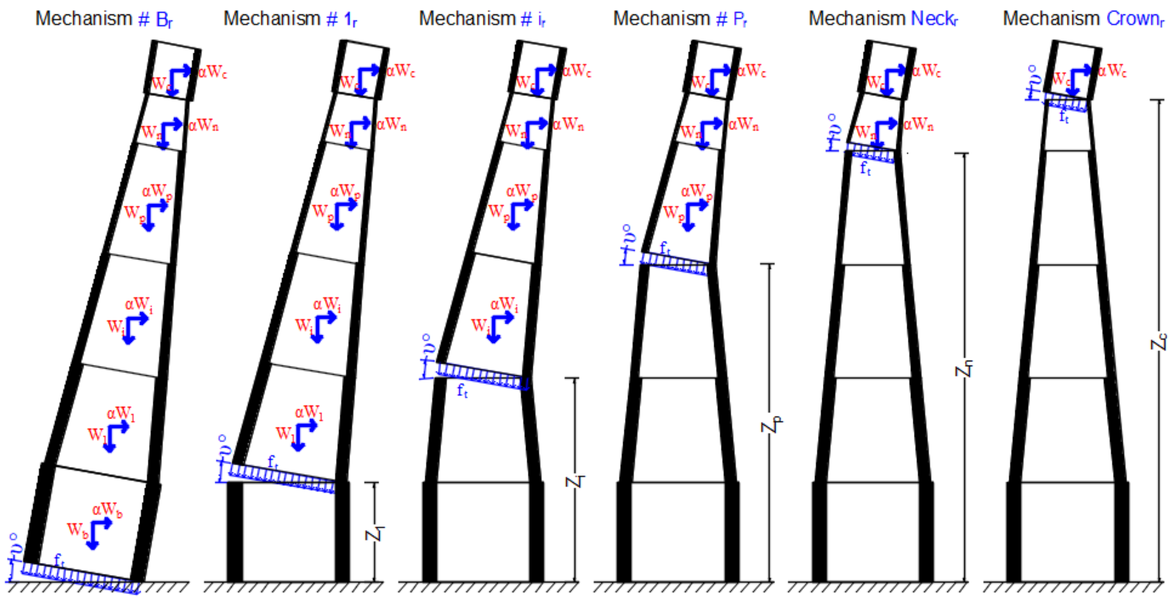


Figure 5: Different rocking failure mechanisms for masonry chimneys based upon the simplified geometry and a limit state approach for the most vulnerable sections.

Local Collapse of the Crown

$$\alpha_{cr} = 2\frac{\left(\frac{b_c}{2} - 0.12\right)(h_c\gamma_M + f_t)}{\gamma_M {h_c}^2} \quad (2)$$

Where with $\gamma_M$we indicate the weight of the masonry material, f_t is the tensile strength here contributing to the overall resistance under lateral loads, and α_{cr} is the load collapse multiplier at the crown of the chimney. The other symbols have been already introduced.

Local Collapse of the Neck

$$\alpha_{nr} = \frac{\left(W_c + W_n + A_{n,base} f_t\right)\frac{b_{n,base}}{2}}{W_c\left(\frac{h_c}{2} + h_n\right) + W_n\left(\frac{A_{n,base} h_n}{A_{n,base} + A_{n,top}}\right)} \tag{3}$$

Where W_c and W_n are the weight of the crown block and neck block respectively estimated as $W_c = A_c h_c \gamma_M$ and $W_n = \left(A_{n,base} + A_{n,top}\right) h_n \gamma_M / 2$, $A_{n,base}$ and $A_{n,top}$ are the areas of the cross section at the base and the top of the block. It can be noted that two indices are used to describe the area, the first one refers to the block and the second one refers to the position of the section, i.e. base or top section. This is done to distinguish different area sections when we pass from one block to another and to describe the truncated pyramidal (conical) geometry of the blocks. The crown and the base are an exception because their shape is cylindrical. α_{nr} is the collapse load multiplier activating a failure mechanism with overturning around a hinge located at the base of the neck.

Local Collapse of the Shaft

A local collapse of the shaft is referred to each failure mechanism that occurs below the neck and above the base of the chimney, i.e., from mechanism #1 to #P in Figure 5. The number of possible failure mechanisms "p" is ruled by the height of each portion of the chimney, total height of the chimney and the height of the intermediate blocks:

$$\alpha_{ir} = \frac{\left(W_c + W_n + \sum_{j=1}^{i} W_j + A_{i,base} f_t\right)\frac{b_{i,base}}{2}}{W_c\left(\frac{h_c}{2} + h_n + (p-i+1)h\right) + W_n\left(\frac{A_{n,base} h_n}{A_{n,base} + A_{n,top}} + (p-i+1)h\right) + \sum_{j=i}^{p} W_j\left((j-i)h + \frac{A_{j,base} h}{A_{j,base} + A_{j,top}}\right)} \tag{4}$$

W_j is the weight of the *j*-th block located above the failure section "i" and it is estimated in the same way as the weight of the neck block. The values of *i* are from *1* to *p*, so we have *p* different collapse load multipliers.

Collapse of the Base

The last failure mechanism considered is the overturning of the whole chimney at the base, and in contrast with the above failure mechanisms, it is not referred as a local one because the hinge is located at the ground level:

$$\alpha_{br} = \frac{\left(W_c + W_n + \sum_{j=1}^{p} W_j + W_b + A_{b,base} f_t\right)\frac{b}{2}}{W_c\left(\frac{h_c}{2} + h_n + p\,h + h_b\right) + W_n\left(\frac{A_{n,base} h_n}{A_{n,base} + A_{n,top}} + p\,h + h_b\right) + \sum_{j=1}^{p} W_j \sum_{j=i}^{p} W_j\left((j-i)h + \frac{A_{j,base} h}{A_{j,base} + A_{j,top}} + h_b\right) + \frac{W_b h_b}{2}} \tag{5}$$

where W_b refers to the weight of the base blocks that can be estimated as: $W_b = A_b h_b \gamma_M$ and all the other symbols have been already introduced.

Sliding Collapse

This failure mechanism is derived from a Mohr-Coulomb failure criterion, expressed in terms of the actual shear stress and vertical compression stress acting in the adjoining section of two blocks. The following expression holds to estimate the collapse multiplier:

$$\alpha_{is} = A_i \frac{\tau_0}{\sum W_j} + \tan \varphi \qquad (6)$$

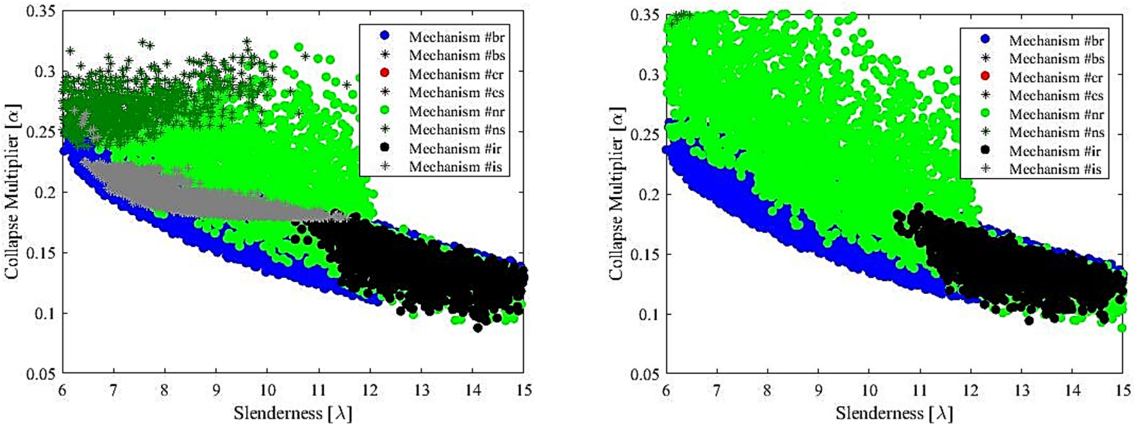
Where A_i is the area of the sliding plane, τ_0 is the cohesion, $\sum W_j$ is the weight of the part of the structure above the sliding plane and φ is the friction angle.

5 PARAMETER INVESTIGATION

Some sensitivity analyses, varying different mechanical parameters aimed at understanding their influence on the prediction of the active failure mechanism, are carried out. By the term "active failure mechanism", we refer to the corresponding failure mechanism which exhibits the minimum collapse load multiplier for a certain combination of the parameters values.

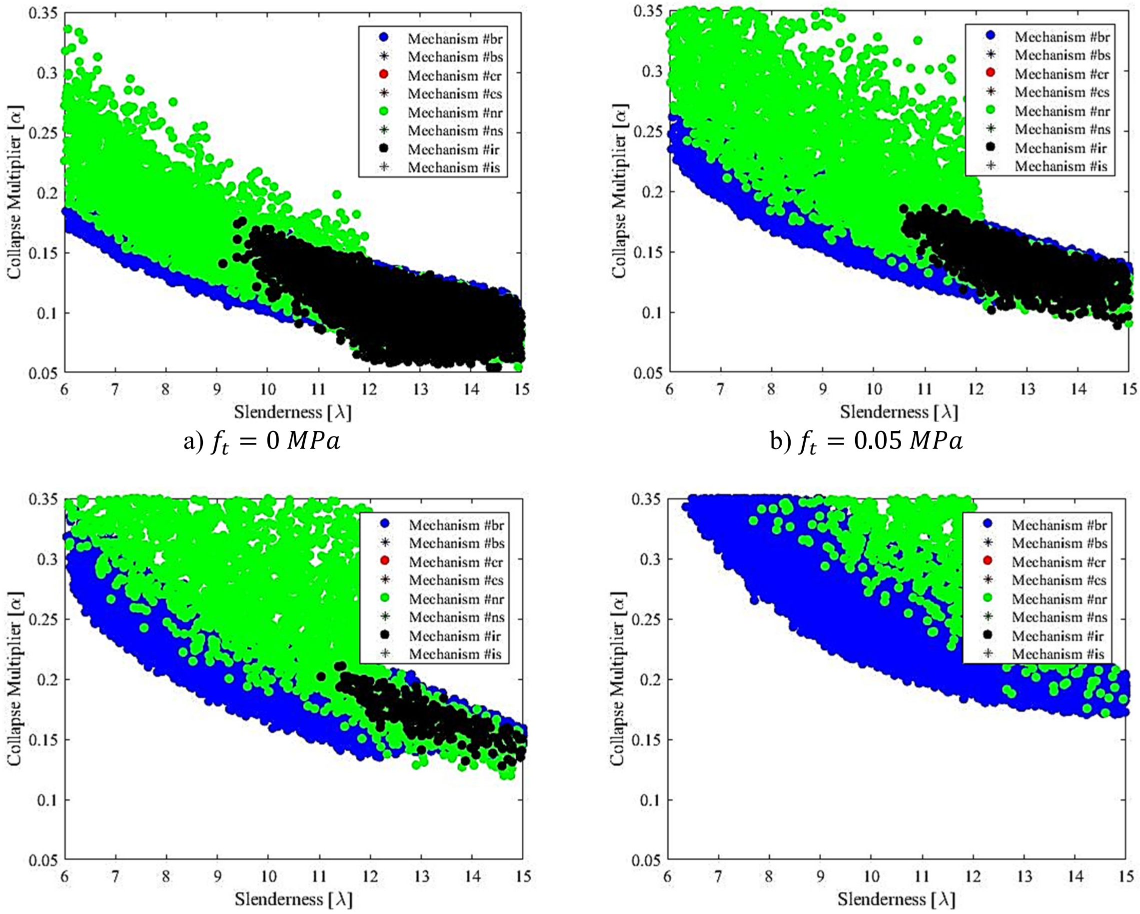
The first investigation is performed varying the friction angle. Figure 6 reports the expected collapse load multiplier and the active failure mechanism for two values of the friction angle, respectively 10° and 15°. In the first case, some failure mechanisms related to shear sliding of the crown, neck, and shaft blocks are active, while in the second case, they are almost absent. Since a reasonable friction angle for masonry is around 30°, it may be concluded that a shear failure mechanism is not much probable to be active in chimneys.

The second investigated parameter is the tensile strength. Four values have been adopted for this study: $f_t = 0$; 0.05; 0.10 ; $0.20\ MPa$ respectively. Figure 7 reports the results obtained in terms of the collapse load multiplier for each considered tensile strength. It can be noted that the tensile strength has a significant role in the vulnerability of a masonry chimney, hence its prior knowledge from suitable experimentation is highly recommended.



a) $\varphi = 10°$ b) $\varphi = 15°$

Figure 6: Collapse load multiplier variability by changing the friction angle of the masonry material.

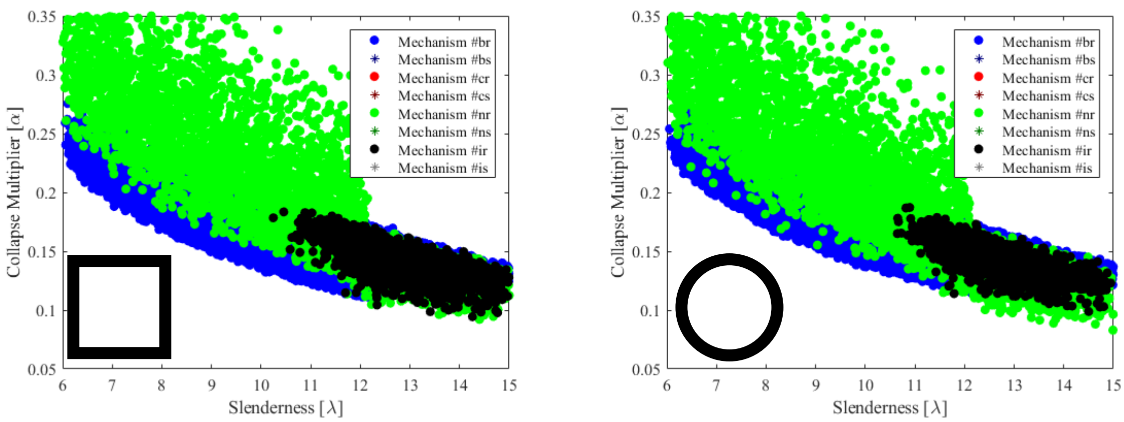


a) $f_t = 0\ MPa$ b) $f_t = 0.05\ MPa$

c) $f_t = 0.1\ MPa$ d) $f_t = 0.2\ MPa$

Figure 7: Collapse load multiplier variability by changing the tensile strength of the masonry material.

The last parameter investigated is the cross-section. As shown in Figure 4, four different cross sections are common for masonry chimneys, namely square, circular, hexagonal and octagonal. Hence the possibility of differing their performance is investigated. Figure 8 presents the collapse load multiplier for all the considered sections, and as can be noted, a not meaningful difference exists.



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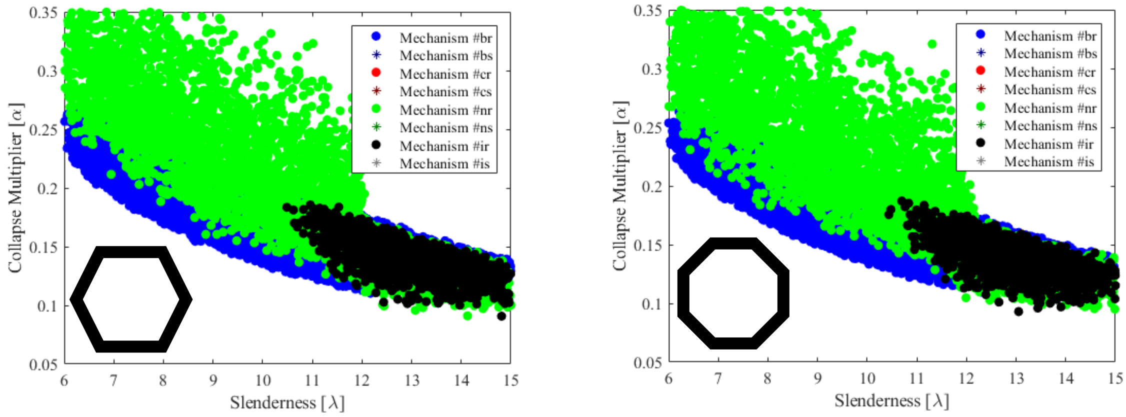
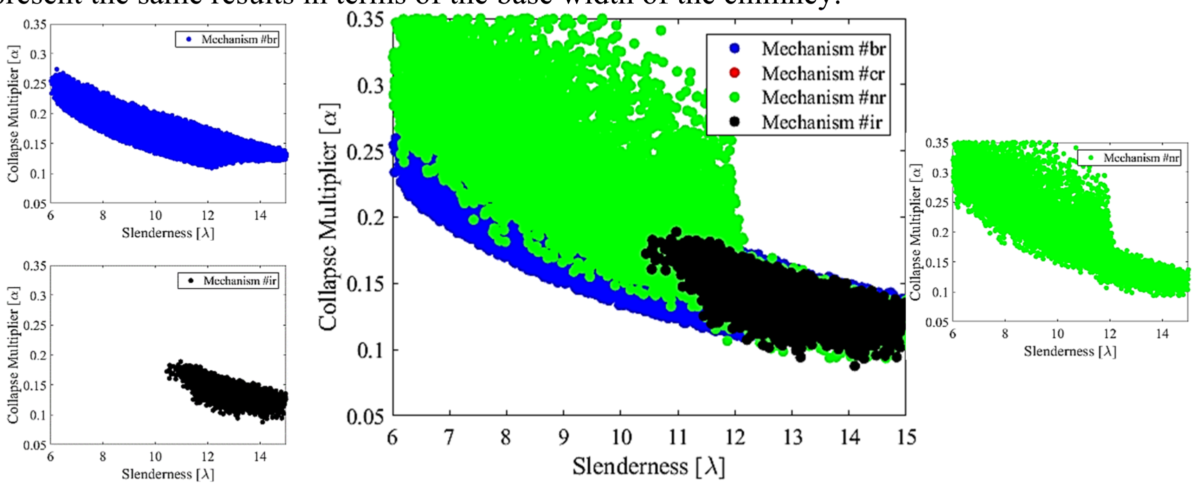


c) d)

Figure 8: Collapse load multiplier variability by changing the cross-section of the chimney.

6 SEISMIC VULNERABILITY CONSIDERATIONS

The above considerations lead to focus attention on failure mechanisms associated to rocking. Among the supposed mechanisms, the crown rocking has very low probability to occur because of the geometrical features, therefore no active mechanism is observed from the numerical simulations. The collapse load multiplier is depicted against slenderness, height and base width of the chimney. In order to distinguish the overlapping graphical effect, a single graph is plotted for each failure mechanism aside from each central image, Figure 9. From Figure 9-a) it can be noticed a change in the trend of the failure mechanism related to the base rocking and neck rocking. It occurs for slenderness roughly higher than 12, and it is followed by a new failure mechanism activated in the shaft blocks. From Figure 9-b), failure of the neck has two distinct branches. The first branch is isolated by a limited height of 15 m which corresponds to the simplest configuration of the chimney, crown-neck-base. The expected collapse multiplier for this branch is very diffused, while for the second branch follows a linear trend. The same trend is also observed for the other mechanisms, i.e., shaft rocking and base rocking. Figure 9-c) present the same results in terms of the base width of the chimney.



a)

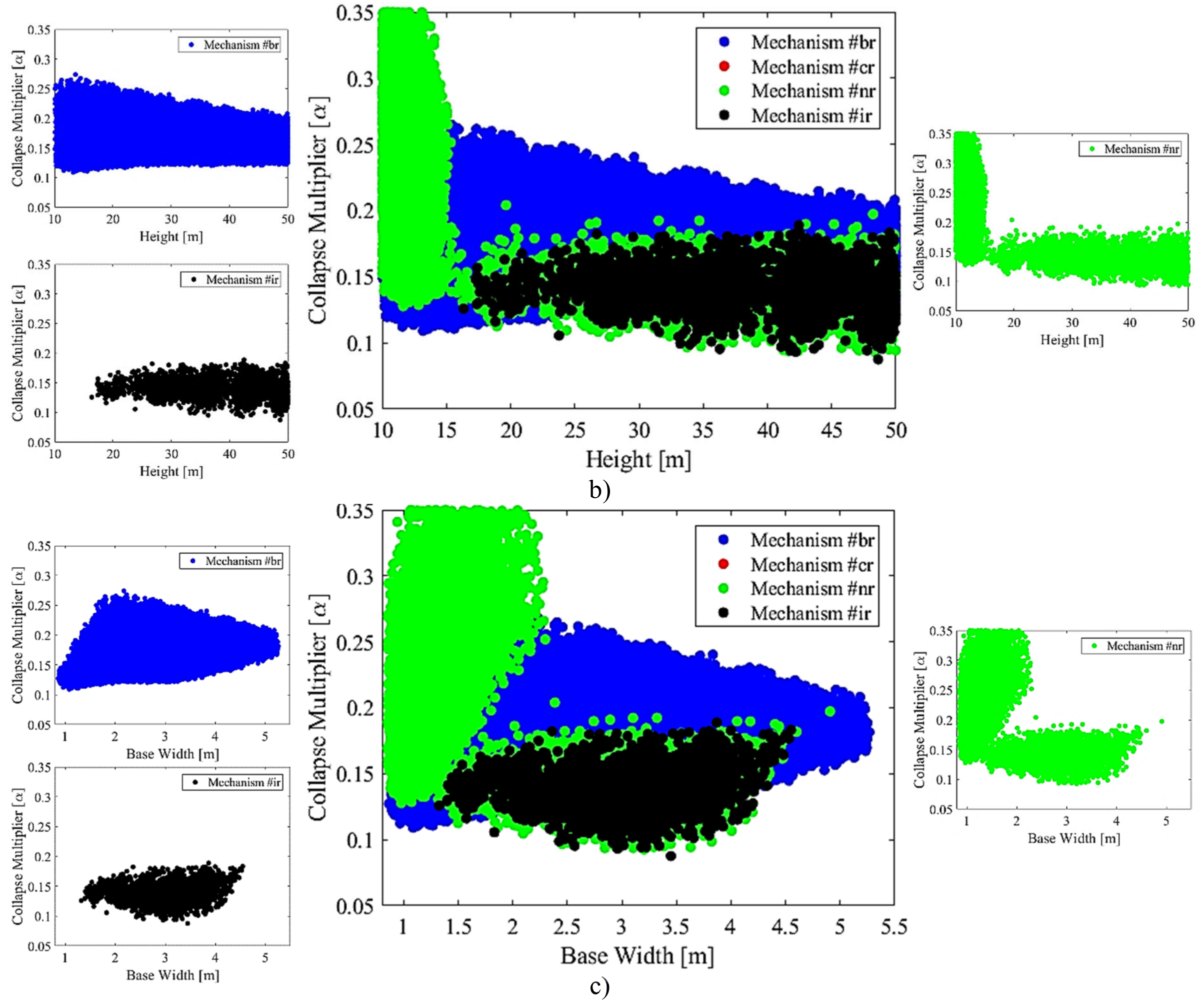


Figure 9: Collapse load multiplier variability respectively by changing the a) Slenderness; b) Height and c) Base width.

The above illustrations are an endeavor to estimate the vulnerability of the masonry chimneys in terms of active failure mechanism and the lowest collapse load multiplier. The numerical results highlight a very high vulnerability of these structures, especially related to the local failures at the upper parts of the structure. From different supposed failure mechanism, four for each rocking motions and sliding one, only three of them are more probable to occur.

7 CONCLUSIONS

This study has presented a simple, general and widely applicable upper bound limit analysis approach to predict the failure mechanism of masonry chimneys subjected to lateral loads. By means of a wide investigation conducted on real case studies on Spain, a simplified parametrization of the chimneys geometry is obtained. An endeavor to study the seismic vulnerability of these parametrized masonry chimneys is carried out by means of sensitive Limit Analysis, changing the value to different involved parameters. From large-scale MC simulations (1000000 combinations), the following conclusions can be drawn.

- The cross-section shape of masonry chimneys does not influence the active failure mechanisms and the expected collapse load multiplier.

- The shear sliding failure mechanisms is quite difficult to be active unless very poor material characteristics are present, e.g., $\varphi < 15°$.
- The tensile strength of the masonry material has a significant role in imposing the active failure mechanism and the corresponding collapse load multiplier. An increment in the tensile strength, tends to activate only the base rocking mechanism and to decrease the overall seismic vulnerability of the chimney.
- For high, slender chimneys, the most probable active failure mechanisms are those known as local failures, where the rocking occur at upper levels of the chimney or above the base block. The last case occurs where the geometry of the base is more massive than the other parts.

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